The Quantum Postulate and the Recent Development of Atomic Theory	量子假设与原子理论的新进展
Author: Niels Henrik David Bohr Time: 1928 Ref: Nature (Supplement) **121**, 580 (1928); Reprinted **6**, 148 (1985). Translators: Sheng Cheng, Mo Ling, Mukai Wang Discussion participants: Zhenqi Chen, Ruihua Ni, Wenhui Zhao Supervisor: Liang Huang Affiliation: Lanzhou Center for Theoretical Physics	作者：尼尔斯·亨利克·戴维·玻尔 时间：1928 文献：Nature (Supplement) **121**, 580 (1928); Reprinted **6**, 148 (1985). 译者：成晟、凌默、王沐恺（姓氏首字母） 参与讨论：陈振齐、倪瑞华、赵文慧 指导老师：黄亮 单位：兰州理论物理中心
In connexion with the discussion of the physical interpretation of the quantum theoretical methods developed during recent years, I should like to make the following general remarks regarding the principles underlying the description of atomic phenomena, which I hope may help to harmonise the different views, apparently so divergent, concerning this subject.	针对近年来量子理论方法的物理解释的相关讨论，我想就描述原子现象[1]所依据的基本原理，提出几点一般性的看法**。**希望这些意见有助于协调该问题上似乎分歧很大的各方观点。 [1] atomic phenomena：原子现象，例如电子跃迁
1. QUANTUM POSTULATE AND CAUSALITY	1. 量子假设与因果性
The quantum theory is characterised by the acknowledgment of a fundamental limitation in the classical physical ideas when applied to atomic phenomena. The situation thus created is of a peculiar nature, since our interpretation of the experimental material rests essentially upon the classical concepts. Notwithstanding the difficulties which hence are involved in the formulation of the quantum theory, it seems, as we shall see, that its essence may be expressed in the so-called quantum postulate, **which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolised by Planck's quantum of action.**	量子理论的一个显著特征是承认将经典物理的思想应用于解释原子现象时存在根本性的局限。由此产生的情形是违反直觉的，因为我们对实验事实的解释，在根本上仍然依赖于经典的物理概念。尽管这种局限性使得量子理论的建立面临诸多困难，但正如我们将会看到的，量子理论的本质似乎可以用所谓的量子假设来加以概括。**这个假设赋予所有原子过程一种本质上的不连续性，或者说分立性，这一点完全不同于经典理论，并且以普朗克的作用量量子[2]为其特征。** [2] Planck's quantum of action：普朗克的作用量量子，其值为普朗克常数h。
This postulate implies a renunciation as regards the causal space-time co-ordination of atomic processes. Indeed, our usual description of physical phenomena is based entirely on the idea that the phenomena concerned may be observed without disturbing them appreciably. This appears, for	这个假设意味着必须放弃对原子过程的因果性时空描述。实际上，我们通常对于物理现象的描述是建立在“观测不会显著地扰动现象本身”这一基础上的。例如，这种观念在相对论中表现得尤为明显，而相

example, clearly in the theory of relativity, which has been so fruitful for the elucidation of the classical theories. **As emphasised by Einstein, every observation or measurement ultimately rests on the coincidence of two independent events at the same space-time point.** Just these coincidences will not be affected by any differences which the space-time co-ordination of different observers otherwise may exhibit. **Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected.** Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. **After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed.** Ultimately every observation can of course be reduced to our sense perceptions. The circumstance, however, that in interpreting observations use has always to be made of theoretical notions, entails that for every particular case it is a question of convenience at what point the concept of observation involving the quantum postulate with its inherent 'irrationality' is brought in.	对论在阐明经典理论方面极具成效。**正如爱因斯坦所强调的，每一次观察或测量，归根结底都是两个独立事件在同一时空点上的重合。**这些所谓的重合，是不会因为不同观察者在不同坐标系观测而不同。然而，**量子假设则意味着，任何对原子现象的观测，都不可避免地导致观测对象与观测装置间发生不可忽略的相互作用。**因此，**通常物理意义上说的独立实在既不能归属于现象，也不能归于观测装置。**归根结底，**观测这一概念本身具有一定的任意性，它取决于我们将哪些对象划入被观测的系统中。**最终所有的观测都需要还原为我们的感官体验。然而，在对观测进行解释时，总是不可避免地使用理论概念，这就意味着，在每一个具体的情形中，何时引入包含量子假设及其内禀的“非理性”的观测概念，实际上是一个视情况而定的问题。
This situation has far-reaching consequences. On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible, and, above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there can be no question of causality in the ordinary sense of the word. The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality, the union of which characterises the classical theories, as complementary but exclusive features of the description, symbolising the idealisation of observation and definition respectively. **Just as the relativity theory has taught us that the convenience of distinguishing sharply between space and time rests solely on the smallness of the velocities ordinarily met with compared to the velocity of light, we learn from the**	这一情况带来了深远的影响。一方面，按照通常的理解，物理系统状态的定义要求排除一切外部干扰。但在这种情况下，根据量子假设，任何观测都是不可能的，最重要的是，空间和时间的概念也将失去其直接的意义。另一方面，如果为了使观测成为可能，我们允许系统与某些不属于该系统的适当测量装置发生某种相互作用，那么对系统状态的明确定义自然就不再可能，也就无法再像通常意义上那样谈论因果性了。因此，量子理论的本质迫使我们把时空描述和因果性要求视为互补又互斥的特性，它们的结合是经典理论的特征，分别象征着对观测的理想化[3]和对系统定义的理想化[4]。**正如相对论告诉我们的，之所以能够清晰地区分空间和时间，只是因为我们通常遇到的速度远小于光速；**同样，**量子理论让我们明白，通常采用的因果性时空描述之所以适用，仅仅是因为普朗克常数相对于日常感知中涉及的作用量来说极其微小。**实际上，在对原

quantum theory that the appropriateness of our usual causal space-time description depends entirely upon the small value of the quantum of action as compared to the actions involved in ordinary sense perceptions. Indeed, in the description of atomic phenomena, the quantum postulate presents us with the task of developing a complementarity theory the consistency of which can be judged only by weighing the possibilities of definition and observation.	子现象的描述中，量子假设使我们必须构建一种互补性理论，这种理论的一致性只有通过权衡对系统的定义与观测才能加以评判。 [3] 对 observation 的理想化：我们可以在不扰动系统的情况下，随时随地测量物体的位置和时间，是 space-time co-ordination 的前提。 [4] 对 definition 的理想化：给系统的状态下一个精确无歧义的定义，包括这里系统里所有的物理量，即为 claim of causality 的前提。
This view is already clearly brought out by the much-discussed question of the nature of light and the ultimate constituents of matter. As regards light, its propagation in space and time is adequately expressed by the electromagnetic theory. Especially the interference phenomena in vacuo and the optical properties of material media are completely governed by the wave theory superposition principle. Nevertheless, the conservation of energy and momentum during the interaction between radiation and matter, as evident in the photoelectric and Compton effect, finds its adequate expression just in the light quantum idea put forward by Einstein. As is well known, the doubts regarding the validity of the superposition principle on one hand and of the conservation laws on the other, which were suggested by this apparent contradiction, have been definitely disproved through direct experiments. This situation would seem clearly to indicate the impossibility of a causal space-time description of the light phenomena. On one hand, in attempting to trace the laws of the time-spatial propagation of light according to the quantum postulate, we are confined to statistical considerations. On the other hand, the fulfilment of the claim of causality for the individual light processes, characterised by the quantum of action, entails a renunciation as regards the space-time description. Of course, there can be no question of a quite independent application of the ideas of space and time and of causality. The two views of the nature of light are rather to be considered as different attempts at an interpretation of experimental evidence in which the limitation of the classical concepts is expressed in complementary ways.	这种互补性的观点，早已在“光的本性”及“物质的最终组成”等广为讨论的问题上表现得十分清楚。就光而言，其在时空中的传播可以用电磁理论充分描述，尤其是真空中的干涉现象和物质介质的光学性质，都完全遵循波动理论的叠加原理。然而，在辐射与物质相互作用过程中能量与动量的守恒，如光电效应和康普顿效应所显示的，却只有通过爱因斯坦提出的光量子概念才能得到恰当的表达。众所周知，针对这一表面上的矛盾所引发的一方面关于叠加原理，另一方面关于守恒定律的有效性的质疑，已被实验彻底否定[5]。这种情形似乎明确地表明，对光现象进行因果性时空描述是不可能的。一方面，在尝试根据量子假设去追溯光的时空传播规律时，我们只能局限于统计学上的讨论；另一方面，若要满足作用量量子所表征的单个光过程的因果性要求，就必须放弃对其时空的直接描述。当然，也就不可能将空间、时间和因果性的观念完全独立地加以应用。对于光的本性，这两种观点应被看作是对实验事实的不同解释尝试，其中，经典概念的局限性以互补的方式得到了体现。 [5] 康普顿-西蒙实验：通过观测单个 X 射线光子与电子的碰撞轨迹，直接证实了能量动量守恒在微观领域同样严格成立；博特-盖格实验：通过符合计数法证实了碰撞过程中能量动量守恒的成立，为守恒定律在量

	子层面的普适性提供了关键证据。
The problem of the nature of the constituents of matter presents us with an analogous situation. The individuality of the elementary electrical corpuscles is forced upon us by general evidence. Nevertheless, recent experience[6], above all the discovery of the selective reflection of electrons from metal crystals, requires the use of the wave theory superposition principle in accordance with the original ideas of L. de Broglie. Just as in the case of light, we have consequently in the question of the nature of matter, so far as we adhere to classical concepts, to face an inevitable dilemma, which has to be regarded as the very expression of experimental evidence. In fact, here again we are not dealing with contradictory but with complementary pictures of the phenomena, which only together offer a natural generalisation of the classical mode of description. In the discussion of these questions, it must be kept in mind that, according to the view taken above, radiation in free space as well as isolated material particles are abstractions, their properties on the quantum theory being definable and observable only through their interaction with other systems. Nevertheless, these abstractions are, as we shall see, indispensable for a description of experience in connexion with our ordinary space-time view.	物质组成性质的问题也向我们呈现出一种类似的处境。大量证据迫使我们不得不承认基本电荷粒子的不可分性。然而，最近的实验结果[6]，特别是电子在金属晶体上选择性反射的发现，又要求我们依据德布罗意的最初设想——借助波动理论的叠加原理来描述。物质的性质问题和光的性质问题一样，只要我们坚持经典物理的概念，就不可避免地要面对一个两难的困境，而这一困境正是实验事实的直接体现。实际上，我们在这里同样面对的，不是相互矛盾而是互补的两种物理图像。只有将它们结合起来，才能对经典模式进行自然的推广。在讨论这些问题时，必须牢记，根据前文所述观点，无论是自由空间中的辐射还是孤立的物质粒子，在量子理论中都只是抽象概念；它们的性质只有通过与其他系统的相互作用才能被定义和观测。尽管如此，正如我们将看到的，这些抽象概念在结合我们通常的时空观念以解释关于物理世界的观测时，依然是不可或缺的。 [6] experience：哲学中指经验事实、实验证据、观测结果，不是单纯的“经验”的意思。
The difficulties with which a causal space-time description is confronted in the quantum theory, and which have been the subject of repeated discussions, are now placed into the foreground by the recent development of the symbolic methods. An important contribution to the problem of a consistent application of these methods has been made lately by Heisenberg (*Zeitschr. f. Phys.*, **43**, 172; 1927). In particular, he has stressed the peculiar reciprocal uncertainty which affects all measurements of atomic quantities. Before we enter upon his results it will be advantageous to show how the complementary nature of the description appearing in this uncertainty is unavoidable already in an analysis of the most elementary concepts employed in interpreting experience.	量子理论中对因果性时空描述所面临的困难，过去已多次成为讨论的话题，而最近符号化方法的发展更使这些问题被突出地摆到了我们面前。最近，海森堡在这些方法的一致性问题上做出了重要贡献(*Zeitschr. f. Phys.*, **43**, 172; 1927)。尤其值得注意的是，他强调了影响所有原子物理量测量的一种特殊的不确定关系。在讨论海森堡的研究成果前需指出，这种不确定性展现的互补性，其实早已蕴含在对理解物理现象所依赖的那些最基本概念的分析之中。

2. QUANTUM OF ACTION AND KINEMATICS

The fundamental contrast between the quantum of action and the classical concepts is immediately apparent from the simple formulae which form the common foundation of the theory of light quanta and of the wave theory of material particles.

If Planck's constant be denoted by h, as is well known,

$$E\tau = I\lambda = h \qquad (1)$$

where E and I are energy and momentum respectively, τ and λ the corresponding period of vibration and wave-length. In these formulae the two notions of light and also of matter enter in sharp contrast. While energy and momentum are associated with the concept of particles, and hence may be characterised according to the classical point of view by definite space-time co-ordinates, the period of vibration and wave-length refer to a plane harmonic wave train of unlimited extent in space and time. Only with the aid of the superposition principle does it become possible to attain a connexion with the ordinary mode of description. Indeed, a limitation of the extent of the wave-fields in space and time can always be regarded as resulting from the interference of a group of elementary harmonic waves. As shown by de Broglie (Thèse, Paris, 1924), the translational velocity of the individuals associated with the waves can be represented by just the so-called group-velocity. Let us denote a plane elementary wave by

$$A\cos 2\pi\left(\nu t - x\sigma_x - y\sigma_y - z\sigma_z + \delta\right),$$

where A and δ are constants determining respectively the amplitude and the phase. The quantity $\nu = I/\tau$ the frequency, σ_x, σ_y, σ_z the wave numbers in the direction of the co-ordinate axes, which may be regarded as vector components of the wave number $\sigma = 1/\lambda$ in the direction of propagation. While the wave or phase velocity is given by ν/σ, the group - velocity is defined by dν/dσ. Now according to the relativity theory we have for a particle with the velocity v:

$$I = \frac{v}{C^2}E \quad \text{and} \quad v\,\mathrm{d}I = \mathrm{d}E$$

where c denotes the velocity of light. Hence by equation (1) the phase velocity is c^2/ν and the group-velocity ν. The circumstance that the former is in general greater than the velocity of light emphasises the symbolic character of these considerations. At the same time, **the possibility of identifying the velocity of the particle with the group-**

2. 作用量量子与运动学

从构成光量子理论与物质波理论共同基础的简单公式中，我们可以直接看出作用量量子与经典物理概念之间的根本差异。如果用表示普朗克常数h，众所周知：

$$E\tau = I\lambda = h \qquad (1)$$

其中E和I分别表示能量和动量，τ和λ分别是对应的振动周期和波长。在这些公式中，描述光的周期和波长与描述物质的能量和动量形成鲜明对比。能量和动量是粒子的相关概念，因此根据经典观点，可以通过明确的时空坐标来描述；而振动周期和波长则对应的是在空间和时间中无限延展的平面简谐波列[7]。只有借助叠加原理，才有可能与经典物理对粒子运动的直观描述建立联系。事实上，在空间和时间中的有限延展的波场[8]，总是可以看作是一组无限延展的基本简谐波彼此干涉的结果。正如德布罗意(Thèse, Paris, 1924)所说的，与波相关的个体（经典粒子）[9]的平动速度正好可以用所谓的群速度来表示。我们用下式表示一个基本平面波，

$$A\cos 2\pi\left(\nu t - x\sigma_x - y\sigma_y - z\sigma_z + \delta\right)$$

其中A和δ分别是决定振幅和初相位的常数。$\nu = 1/\tau$代表频率，σ_x，σ_y，σ_z是沿坐标轴方向的波数，可以看作是沿传播方向的总波数 $\sigma = 1/\lambda$ 的各个分量。波的相速度由 ν/σ 给出，群速度则定义为$d\nu/d\sigma$。根据相对论理论，对于速度为v的粒子，有：

$$I = \frac{v}{c^2}E \text{ 且 } v\,\mathrm{d}I = \mathrm{d}E$$

其中c表示光速。因此根据公式（1），相速度为c^2/ν ，而群速度为ν。前者通常大于光速，这一事实显示了其仅为形式化的数学推导，并无直接的物理实在对应。与此同时，**将粒子的速度与群速度等同的可能性，指明了在量子理论中使用时空图像的适用范围。**在这里，描述的互补性特征显现出来，因为很多平面波叠加出来的波包的使用必然伴随着对周期和波长定义的不精确性，从而也影响到由公式（1）给出的能量和动量的定义。

[7] a plane harmonic wave train：这里指的是无限延展的平面简谐波，是纯粹

velocity indicates the field of application of space-time pictures in the quantum theory. Here the complementary character of the description appears, since the use of wave-groups is necessarily accompanied by a lack of sharpness in the definition of period and wave-length, and hence also in the definition of the corresponding energy and momentum as given by relation (1).	波动性（具有明确波长和周期、但无法定位）的理想化模型。 [8] wave-fields: 这是薛定谔引入波动力学前的通用术语，可以认为是波函数。在强调有限延展的 wave-fields 时，它与波包相同。 [9] individual：表示“个体”，玻尔在文章中使用这个词同时表示“经典粒子”和“量子客体”，意在暗指这些不精确的“量子客体”，正是能够描述“经典粒子”的东西。
Rigorously speaking, a limited wave-field can only be obtained by the superposition of a manifold of elementary waves corresponding to all values of ν and σ_x, σ_y, σ_z. But the order of magnitude of the mean difference between these values for two elementary waves in the group is given in the most favourable case by the condition $\Delta t \Delta \nu = \Delta x \Delta \sigma_x = \Delta y \Delta \sigma_y = \Delta z \Delta \sigma_z = 1$ where Δt, Δx, Δy, Δz denote the extension of the wave-field in time and in the directions of space corresponding to the co-ordinate axes. These relations—well known from the theory of optical instruments, especially from Rayleigh's investigation of the resolving power of spectral apparatus—express the condition that the wave-trains extinguish each other by interference at the space-time boundary of the wave-field. They may be regarded also as signifying that the group as a whole has no phase in the same sense as the elementary waves. From equation (1) we find thus : $\Delta t \Delta E = \Delta x \Delta I_x = \Delta y \Delta I_y = \Delta z \Delta I_z = h$ (2) as determining the highest possible accuracy in the definition of the energy and momentum of the individuals associated with the wave-field. In general, the conditions for attributing an energy and a momentum value to a wave-field by means of formula (1) are much less favourable. Even if the composition of the wave-group corresponds in the beginning to the relations (2), it will in the course of time be subject to such changes that it becomes less and less suitable for representing an individual. It is this very circumstance which gives rise to the paradoxical character of the problem of the nature of light and of material particles. The limitation in the classical concepts expressed through relation (2) is, besides, closely connected with the	严格来说，一个受限的波场只能通过对所有ν以及σ_x，σ_y，σ_z取值的基本波进行叠加而得到。在最理想情形下，波包中任意两列基本波的参数之间的平均差异满足： $\Delta t \Delta \nu = \Delta x \Delta \sigma_x = \Delta y \Delta \sigma_y = \Delta z \Delta \sigma_z = 1$ 其中，Δt，Δx，Δy，Δz分别表示波场在时间和空间各坐标轴方向上的延展范围，即它的时空尺度。这些关系式在光学仪器理论中早已为人熟知，尤其是在瑞利关于光谱仪分辨能力的研究中。它们可以实现这样一个条件：波列在受限波场的时空延展边界处通过干涉而彼此抵消。这些关系还可以理解为，整个波包作为一个整体，并不像基本波（平面波）那样具有相同意义上的相位。从方程（1）我们可以得到： $\Delta t \Delta E = \Delta x \Delta I_x = \Delta y \Delta I_y = \Delta z \Delta I_z = h$ （2） 这等价于确定了与波场相关的个体（量子客体）的能量和动量的最高可能精度。一般而言，公式（1）为波场赋予的能量和动量的条件远没有这么理想。即使波包在初始时刻满足关系式（2），随着时间推移，它也会发生变化，使得它越来越不适合用来描述一个个体（经典粒子）。正是这种情况导致了光和物质粒子本质问题的悖论性质。通过关系式（2）表达的经典概念的局限性，还与经典力学的有限适用性密切相关。在物质的波动理论中，这种有限性对应于几何光学，在几何光学中，波的传播被描述为“光线”。只有在这一极限下，才能基于时空图像对能量和动量做出明确的定义。对于这些概念的一般性定义，我们只能依

limited validity of classical mechanics, which in the wave theory of matter corresponds to the geometrical optics, in which the propagation of waves is depicted through 'rays'. Only in this limit can energy and momentum be unambiguously defined on the basis of space-time pictures. For a general definition of these concepts we are confined to the conservation laws, the rational formulation of which has been a fundamental problem for the symbolical methods to be mentioned below.	赖守恒定律，而这些定律的合理表述是下文将要提到的符号化方法一直要处理的一个基本问题[10]。 [10] 在非几何光学近似下，时空图像失效，定义能量、动量需基于守恒律，其量子化表述是矩阵/波动力学等符号化方法的核心任务之一。
In the language of the relativity theory, the content of the relations (2) may be summarised in the statement that according to the quantum theory a general reciprocal relation exists between the maximum sharpness of definition of the space-time and energy-momentum vectors associated with the individuals. This circumstance may be regarded as a simple symbolical expression for the complementary nature of the space-time description and the claims of causality. At the same time, however, the general character of this relation makes it possible to a certain extent to reconcile the conservation laws with the space-time coordination of observations, the idea of a coincidence of well-defined events in a space-time point being replaced by that of unsharply defined individuals within finite space-time regions.	用相对论的语言来说，关系式（2）的内容可以总结为：量子理论中，个体（量子客体）的时空坐标最高测量精度与其能动矢量最高测量精度之间，存在普适的倒数型制约关系。这种情况可以被看作是时空描述的互补性与因果性要求的一个简明的符号化表达。然而，这一关系的普遍性质，使得我们能够在某种程度上实现守恒定律与时空描述之间的相容。此时，经典物理中在时空点上明确定义事件的观念，被替换为在有限时空区域内定义不精确的个体（量子客体）的观念。
This circumstance permits us to avoid the well-known paradoxes which are encountered in attempting to describe the scattering of radiation by free electrical particles as well as the collision of two such particles. According to the classical concepts, the description of the scattering requires a finite extent of the radiation in space and time, while in the change of the motion of the electron demanded by the quantum postulate one seemingly is dealing with an instantaneous effect taking place at a definite point in space. Just as in the case of radiation, however, it is impossible to define momentum and energy for an electron without considering a finite space-time region. Furthermore, **an application of the conservation laws to the process implies that the accuracy of definition of the energy momentum vector is. the same for the radiation and the electron. In consequence, according to relation (2), the associated space-time regions can be given the same size for both individuals in interaction.**	这种情况使我们能够避免那些在试图描述自由带电粒子对光子的散射以及两个此类粒子的碰撞时所遇到的著名悖论。根据经典概念，描述散射过程需要光子在空间和时间上具有有限的延展性；而根据量子假设，电子运动的改变似乎是发生在空间中某一确定点上的瞬时效应。然而，正如在光子的情形中一样，如果仅用时空点而不考虑有限的时空区域，就无法给电子定义动量和能量。此外，**将守恒定律应用于这一过程，还意味着光量子和电子的能动矢量的定义精度是相同的。因此，根据关系式（2），相互作用的两个个体（量子客体）所对应的时空区域具有同样的大小[11]。** [11] 相互作用中，涉及到的两个个体的时空尺度受同一个不确定性关系约束。

A similar remark applies to the collision between two material particles, although the significance of the quantum postulate for this phenomenon was disregarded before the necessity of the wave concept was realised. Here this postulate does indeed represent the idea of the individuality of the particles which, transcending the space-time description, meets the claim of causality. While the physical content of the light quantum idea is wholly connected with the conservation theorems for energy and momentum, in the case of the electrical particles the electric charge has to be taken into account in this connexion. It is scarcely necessary to mention that for a more detailed description of the interaction between individuals we cannot restrict ourselves to the facts expressed by formula (1) and (2), but must resort to a procedure which allows us to take into account the coupling of the individuals, characterising the interaction in question, where just the importance of the electric charge appears. As we shall see, such a procedure necessitates a further departure from visualisation in the usual sense.	类似的论述也适用于两个物质粒子的碰撞，尽管在认识到波动概念的必要性之前，这一现象中量子假设的重要性曾被忽视。在这里，这一假设确实体现了粒子个体性的思想，这种个体性超越了时空描述[12]，满足了因果性的要求。光量子思想的物理内容完全与能量和动量的守恒定理相关，而对电子等带电粒子而言，必须将电荷也纳入考虑。为了更详细地描述个体之间的相互作用，我们不能仅仅满足于公式（1）和（2）所表达的事实，而必须采用一种能够考虑到个体之间耦合、并能够刻画所讨论相互作用的过程，其中电荷的重要性正是在此体现出来的。正如我们将要看到的，这样的处理方法必然要求我们进一步超越通常意义上的直观图像。 [12] “超越了时空描述”：之前经典物理中的个体性体现在不同的时空轨迹上，个体性完全依赖时空描述；但现在抛开精确的时空描述，个体性依然存在。“满足了因果性的要求”是指守恒律在微观相互作用中依然成立。

3. MEASUREMENTS IN THE QUANTUM THEORY	3. 量子理论中的测量
In his investigations already mentioned on the consistency of the quantum theoretical methods, Heisenberg has given the relation (2) as an expression for the maximum precision with which the space-time co-ordinates and momentum-energy components of a particle can be measured simultaneously. His view was based on the following consideration: On one hand, the co-ordinates of a particle can be measured with any desired degree of accuracy by using, for example, an optical instrument, provided radiation of sufficiently short wave-length is used for illumination. According to the quantum theory, however, the scattering of radiation from the object is always connected with a finite change in momentum, which is the larger the smaller the wave-length of the radiation used. The momentum of a particle, on the other hand, can be determined with any desired degree of accuracy by measuring, for example, the Doppler effect of the scattered radiation, provided the wave-length of the radiation is so large that the effect of recoil can be neglected, but then the determination of the space co-ordinates of the particle becomes correspondingly less accurate.	在已经提到过的量子理论方法一致性的研究中，海森堡提出了关系式（2），用来描述一个粒子的时空坐标与能动量分量被同时测量时能够达到的最大精度。他的观点基于如下思考：一方面，例如借助光学仪器，只要使用波长足够短的辐射进行照明，就可以以任意高的精度测量粒子的时空坐标。然而，根据量子理论，物体导致的光子散射总会导致一定的动量变化，而且所用光子的波长越短，这种动量变化就越大；另一方面，只要所用光子的波长长到可以忽略反冲效应，就可以通过测量散射光子的多普勒效应，以任意高的精度确定粒子的动量。但在这种情况下，对粒子空间坐标的测定就会相应地变得不够精确。 注：以康普顿散射为例，在某一θ的散射角度上，对应的光子动量变化为 $$\Delta p = h\sqrt{\frac{1}{\lambda^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda\lambda'}}$$ 其中，$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta)$
The essence of this consideration is the inevitability of the quantum postulate in the estimation of the possibilities of measurement. A closer investigation of the possibilities of definition would still seem necessary in order to bring out the general complementary character of the description. Indeed, a discontinuous change of energy and momentum during observation could not prevent us from ascribing accurate values to the space-time co-ordinates, as well as to the momentum-energy components before and after the process. The reciprocal uncertainty which always affects the values of these quantities is, as will be clear from the preceding analysis, **essentially an outcome of the limited accuracy with which changes in energy and momentum can be defined, when the wave-fields used for the determination of the space-time co-ordinates of the particle are sufficiently small.**	这一考量的本质在于评估测量的可能性时发现量子假设是不可避免的。想要揭示描述方式所具有的普遍的互补性的特征仍然需要进一步深入考察各种方式的可能性。诚然，观测过程中能量与动量有不连续的变化，这并不妨碍我们在过程发生前后，将时空坐标及能动量的分量赋予明确的数值。正如前文所说，这些物理量的数值总受到不确定性关系的制约。这种不确定性源于，**当用来确定粒子时空坐标的波场被限制得足够小时，其所能定义的能量和动量变化的精度就会受到根本性的限制。**
In using an optical instrument for determinations of position, it is necessary to remember that the formation of the image always requires a convergent beam of light.	在利用光学仪器测量位置时，需要记得成像过程必定依赖于一束会聚的光线。若用λ表示所用辐射的波长，用E表示所谓的数值

Denoting by λ the wave-length of the radiation used, and by E the so-called numerical aperture, that is, the sine of half the angle of convergence, the resolving power of a microscope is given by the well-known expression $\lambda/2E$. Even if the object is illuminated by parallel light, so that the momentum h/λ, of the incident light quantum is known both as regards magnitude and direction, the finite value of the aperture will prevent an exact knowledge of the recoil accompanying the scattering. Also, even if the momentum of the particle were accurately known before the scattering process, our knowledge of the component of momentum parallel to the focal plane after the observation would be affected by an uncertainty amounting to $2Eh/\lambda$. The product of the least inaccuracies with which the positional co-ordinate and the component of momentum in a definite direction can be ascertained is therefore just given by formula (2). One might perhaps expect that in estimating the accuracy of determining the position, not only the convergence but also the length of the wave-train has to be taken into account, because the particle could change its place during the finite time of illumination. Due to the fact, however, that the exact knowledge of the wave-length is immaterial for the above estimate, it will be realised that for any value of the aperture the wave-train can always be taken so short that a change of position of the particle during the time of observation may be neglected in comparison to the lack of sharpness inherent in the determination of position due to the finite resolving power of the microscope.	孔径，即会聚角一半的正弦值，则显微镜的分辨率由著名公式 $\lambda/2E$ 所给出。即便物体被平行光照明，入射光量子的动量h/λ无论在大小还是方向上都已知，有限的孔径仍会使我们无法准确得知散射伴随的反冲的情形。同理，即使粒子在散射前的动量已知，观测后其在焦平面方向上的动量分量，仍有$2Eh/\lambda$量级的不确定度的影响。因此，位置坐标与某一确定方向上动量分量的不确定度乘积，恰好由公式（2）所示。或许有人会认为，在估算位置测定的精度时，除会聚性之外，还应当考虑波列的长度，因为在有限的照明时间内，粒子的位置可能会发生变化。然而，由于上述估算并不依赖于对波长的精确值，我们可以认为对于任意孔径值，波列的持续时间总可以取得足够短，以至于相较于显微镜分辨本领有限性所带来的定位模糊而言，可以忽略粒子在观测时间内位置的变化。
In measuring momentum with the aid of the Doppler effect—with due regard to the Compton effect—one will employ a parallel wave-train. For the accuracy, however, with which the change in wave-length of the scattered radiation can be measured the extent of the wave-train in the direction of propagation is essential. If we assume that the directions of the incident and scattered radiation are parallel and opposite respectively to the direction of the position co-ordinate and momentum component to be measured, then $c\lambda/2l$ can be taken as a measure of the accuracy in the determination of the velocity, where denotes the length of the wave-train. For simplicity, we here have regarded the velocity of light as large compared to the velocity of the particle. If m represents the mass of the	若利用多普勒效应并兼顾康普顿效应来测定动量，则会使用平行波列。然而，测定散射光子波长变化的精度取决于波列在传播方向上的长度。若设入射与散射光子的方向分别与所测位置坐标和动量分量的方向平行和相反，则 $c\lambda/2l$ 可作为速度测量精度的度量，其中 l 表示波列的长度。为简明起见，此处假定光速远大于粒子的速度。若以 m 表示粒子的质量，则观测后动量的不确定度为 $cm\lambda/2l$。在这种情况下，反冲量的大小明确为$2h/\lambda$，因此不会对粒子观测后动量的取值造成显著的不确定性。实际上，康普顿效应的一般理论告诉我们用入射与散射光子的波长可以计算到反冲前后

particle, then the uncertainty attached to the value of the momentum after observation is $cm\lambda/2l$. In this case the magnitude of the recoil, $2h/\lambda$, is sufficiently well defined in order not to give rise to an appreciable uncertainty in the value of the momentum of the particle after observation. Indeed, the general theory of the Compton effect allows us to compute the momentum components in the direction of the radiation before and after the recoil from the wavelengths of the incident and scattered radiation. Even if the positional co-ordinates of the particle were accurately known in the beginning, our knowledge of the position after observation nevertheless will be affected by an uncertainty. Indeed, on account of the impossibility of attributing a definite instant to the recoil, we know the mean velocity in the direction of observation during the scattering process only with an accuracy $2h/m\lambda$. The uncertainty in the position after observation hence is $2hl/mc\lambda$. Here, too, the product of the inaccuracies in the measurement of position and momentum is thus given by the general formula (2).	光子方向上的动量分量。即便粒子初始的位置坐标已知，观测结束后对其位置的了解，依然会受到不确定性的影响。这是因为我们无法确定反冲的确切时刻，在散射过程中，我们对沿观测方向的平均速度的了解，只能达到$2h/m\lambda$的精度。因此，观测后粒子位置的不确定度为$2hl/mc\lambda$。由这个例子可知，位置与动量测定误差的乘积，也由通用公式（2）给出。
Just as in the case of the determination of position, the time of the process of observation for the determination of momentum may be made as short as is desired if only the wavelength of the radiation used is sufficiently small. The fact that the recoil then gets larger does not, as we have seen, affect the accuracy of measurement. It should further be mentioned, that in referring to the velocity of a particle as we have here done repeatedly, the purpose has only been to obtain a connexion with the ordinary space-time description convenient in this case. **As it appears already from the considerations of de Broglie mentioned above, the concept of velocity must always in the quantum theory be handled with caution.** It will also be seen that **an unambiguous definition of this concept is excluded by the quantum postulate.** This is particularly to be remembered when comparing the results of successive observations. Indeed, the position of an individual at two given moments can be measured with any desired degree of accuracy; but if, from such measurements, we would calculate the velocity of the individual in the ordinary way, it must be clearly realised that we are dealing with an abstraction, from which no unambiguous information concerning the previous or future behaviour of the individual can be obtained.	正如在测定位置时的情况一样，若所用光子的波长足够短，则动量测量所需的观测时间亦可任意缩短。正如我们已经看到的那样，反冲因此变得更大，但这并不影响测量的精度。进一步指出，我们在论述中多次提及粒子速度，目的仅在于方便地与通常的时空描述联系起来。**正如前文所引德布罗意的理论，速度这一概念在量子理论中必须始终谨慎使用。**事实上，**量子假设本身无法对速度作出明确的定义。**这一点在比较连续两次观测结果时尤其需要注意。事实上，某一个体在两个特定时刻的位置可被测定至任意精度；但如果我们以通常的方式用这些测量来计算粒子的速度，则必须清楚意识到，这种方法得到的速度仅为一种抽象概念，无法由此获得关于该个体过去或未来行为的确切信息。

According to the above considerations regarding the possibilities of definition of the properties of individuals, it will obviously make no difference in the discussion of the accuracy of measurements of position and momentum of a particle if collisions with other material particles are considered instead of scattering of radiation. In both cases we see that the uncertainty in question equally affects the description of the agency of measurement and of the object. In fact, this uncertainty cannot be avoided in a description of the behaviour of individuals with respect to a co-ordinate system fixed in the ordinary way by means of solid bodies and unperturbable clocks. The experimental devices—opening and closing of apertures, etc.—are seen to permit only conclusions regarding the space-time extension of the associated wave-fields.	根据前文所述关于个体性质是否可明确定义的讨论，显然，若将粒子与其他物质粒子的碰撞代替光子的散射来观测粒子的位置与动量的测量精度，结果也是一样的。这两种情形中，我们都可以发现，这种不确定性同样影响测量装置与被测对象的描述。事实上，若以通常方式，借助用作空间基准的刚性尺和用作时间基准的理想时钟构建的坐标系来描述个体的行为，这种不确定性便不可避免。实验装置（如孔径的开闭等）所展示的结果仅能得出关于相关波场在时空上的延展情况的结论。
In tracing observations back to our sensations, once more regard has to be taken to the quantum postulate in connexion with the perception of the agency of observation, be it through its direct action upon the eye or by means of suitable auxiliaries such as photographic plates, Wilson clouds, etc. It is easily seen, however, that the resulting additional statistical element will not influence the uncertainty in the description of the object. It might even be conjectured that the arbitrariness in what is regarded as object and what as agency of observation would open up a possibility of avoiding this uncertainty altogether. In connexion with the measurement of the position of a particle, one might, for example, ask whether the momentum transmitted by the scattering could not be determined by means of the conservation theorem from a measurement of the change of momentum of the microscope — including light source and photographic plate — during the process of observation. A closer investigation shows, however, that such a measurement is impossible, if at the same time one wants to know the position of the microscope with sufficient accuracy. In fact, it follows from the experiences which have found expression in the wave theory of matter, that the position of the centre of gravity of a body and its total momentum can only be defined within the limits of reciprocal accuracy given by relation (2).	当我们将观测追溯至自身感官时，仍然需要考虑量子假设与观测装置被感知的方式之间的关系。无论是观测装置直接作用于眼睛，或者是借助诸如感光板、威尔逊云室等辅助器材，都是一样的。然而，不难看出，由此引入的附加统计性因素，并不会影响被测对象在描述中的不确定性。或许有人会想，究竟哪些被视为对象、哪些被视为观测装置，这个界定是否存有任意性。这种划分上的自由，似乎为彻底消除这种不确定性提供了可能。以粒子位置测量为例，有人或许会提出，能否通过测量整个显微镜（包括光源与感光板）在观测过程中动量的变化，并借助守恒定律，来推知散射过程传递的动量？然而，深入考察即可发现，如果要求在测量动量变化的同时，也能精确获知显微镜的位置，此类测量根本无法实现。事实上，正如物质波理论所揭示的，物体重心的位置与其总动量只能在由公式（2）所规定的互为约束的精度范围内被定义。

Strictly speaking, the idea of observation belongs to the causal space-time way of description. Due to the general character of relation (2), however, this idea can be consistently utilised also in the quantum theory, if only the uncertainty expressed through this relation is taken into account. As remarked by Heisenberg, one may even obtain an instructive illustration to the quantum theoretical description of atomic (microscopic) phenomena by comparing this uncertainty with the uncertainty, due to imperfect measurements, inherently contained in any observation as considered in the ordinary description of natural phenomena. He remarks on that occasion that even in the case of macroscopic phenomena we may say, in a certain sense, that they are created by repeated observations. It must not be forgotten, however, that **in the classical theories any succeeding observation permits a prediction of future events with ever-increasing accuracy, because it improves our knowledge of the initial state of the system. According to the quantum theory, just the impossibility of neglecting the interaction with the agency of measurement means that every observation introduces a new uncontrollable element.** Indeed, it follows from the above considerations that the measurement of the positional coordinates of a particle is accompanied not only by a finite change in the dynamical variables, but also the fixation of its position means a complete rupture in the causal description of its dynamical behaviour, while the determination of its momentum always implies a gap in the knowledge of its spatial propagation. Just this situation brings out most strikingly the complementary character of the description of atomic phenomena which appears as an inevitable consequence of the contrast between the quantum postulate and the distinction between object and agency of measurement, inherent in our very idea of observation.	严格而言，“观测”这一概念，原本是因果时空中的描述方式。然而，鉴于不确定性关系的普适性，只要将该式所表达的不确定性纳入考量，观测概念在量子理论中也可自洽地加以应用。正如海森堡指出的，把量子论中这种不确定性，与经典自然现象的描述中因不完善测量所导致的固有的不确定性作比较，还可获得对原子现象的量子描述的有益启示。他还曾指出，即使在宏观现象中，从某种意义上说，这些现象也是由不断观测所“创造”出来的。然而，不能忘记的是，**在经典理论中，任何进一步的观测都能令我们对未来事件的预测变得愈发精确，因为每次观测都会完善我们对系统初始状态的了解。而按量子理论，正因无法忽略与观测装置之间的相互作用，每次观测都必然引入一个新的、不可控制的因素。**像之前所讨论的，测量粒子的位置坐标不仅会带来动力学变量的一定变化，对其位置的确定更意味着割裂了对其动力学演化的因果描述；而对其动量的测定则总会导致对粒子空间传播情形的认识留下空白[13]。这种测量情境最清晰地揭示出，原子现象的互补描述，实际上是“量子假设”与“区分对象和仪器”冲突的必然结果。而这种分离观念，正是我们理解“观测”的认知基。 [13] 位置测量会破坏动力学历史（因果性），动量测量会破坏空间路径（时空描述）

4. CORRESPONDENCE PRINCIPLE AND MATRIX THEORY	4. 对应原理与矩阵理论
Hitherto we have only regarded certain general features of the quantum problem. The situation implies, however, that the main stress has to be laid on the formulation of the laws governing the interaction between the objects which we symbolise by the abstractions of isolated particles and radiation. Points of attack for this formulation are presented in the first place by the problem of atomic constitution. As is well known, it has been possible here, by means of an elementary use of classical concepts and in harmony with the quantum postulate, to throw light on essential aspects of experience. For example, **the experiments regarding the excitation of spectra by electronic impacts and by radiation are adequately accounted for on the assumption of discrete stationary states and individual transition processes. This is primarily due to the circumstance that in these questions no closer description of the space-time behaviour of the processes is required.**	到目前为止，我们只考虑了量子问题的某些一般特征。然而，这种情况意味着，主要的精力必须放在支配物体之间相互作用规律的表述上，这些物体是用孤立的粒子和辐射等抽象概念所符号化的。原子结构的问题是这种理论构建的首要切入口。众所周知，在这里，通过对经典概念的基本使用，并结合量子假设，可以阐明许多实验现象的本质。例如，**关于电子碰撞和辐射激发光谱的实验，假定存在离散定态和个体跃迁过程，就可以得到充分的解释。这主要是因为在这些问题中，不需要对过程的时空行为进行更详细的描述。**
Here the contrast with the ordinary way of description appears strikingly in the circumstance that spectral lines, which on the classical view would be ascribed to the same state of the atom, will, according to the quantum postulate, correspond to separate transition processes, between which the excited atom has a choice. Notwithstanding this contrast, however, a formal connexion with the classical ideas could be obtained in the limit, where the relative difference in the properties of neighbouring stationary states vanishes asymptotically and where in statistical applications the discontinuities may be disregarded. Through this connexion it was possible to a large extent to interpret the regularities of spectra on the basis of our ideas about the structure of the atom.	在光谱线问题上，量子理论与传统描述方式之间的对比显得尤为突出，在经典观点中，多条谱线应归于原子的同一状态。而根据量子假设，每条谱线将对应于单独的跃迁过程，受激原子可以在这些跃迁过程之间进行选择。尽管有这种对比，但在极限情况下仍可以获得与经典形式上的联系，即当相邻定态性质的相对差异趋于零，并且在统计处理时可以忽略能级间的不连续性。通过这种联系，我们很大程度上可以基于原子结构的观念来解释光谱的规律。
The aim of regarding the quantum theory as a rational generalisation of the classical theories led to the formulation of the so-called correspondence principle. The utilisation of this principle for the interpretation of spectroscopic results was based on a symbolical application of classical electrodynamics, in which the individual transition processes were each associated with a harmonic in the motion of the atomic particles to be expected according to ordinary mechanics. Except in the limit mentioned, where the relative difference between adjacent stationary states may be neglected, such a	将量子理论视为经典理论的合理推广的这一目的导致了所谓的对应原理的构建。该原理在光谱学解释中的应用，实际上是一种对经典电动力学的符号化运用。在这种情况下，每个单独的跃迁过程都与根据普通力学预期的原子粒子运动中的某个谐波相关联。除了上述极限情况，即相邻定态之间的相对差异可以忽略不计的情况外，这种对经典理论的零碎应用只能在特定情况下对现象进行严

fragmentary application of the classical theories could only in certain cases lead to a strictly quantitative description of the phenomena. Especially the connexion developed by Ladenburg and Kramers between the classical treatment of dispersion and the statistical laws governing the radiative transition processes formulated by Einstein should be mentioned here. Although it was just Kramers' treatment of dispersion that gave important hints for the rational development of correspondence considerations, it is only through the quantum theoretical methods created in the last few years that the general aims laid down in the principle mentioned have obtained an adequate formulation.	格定量的描述。特别值得一提的是拉登堡和克拉默斯在色散的经典处理和爱因斯坦建立的支配辐射跃迁过程的统计定律之间建立了联系。虽然正是克拉默斯对色散问题的处理为对应原理的合理发展提供了重要线索，但直到最近几年建立的量子理论方法的出现，这一原理所确立的总体目标才得以充分实现。
As is known, the new development was commenced in a fundamental paper by Heisenberg, where he succeeded in emancipating himself completely from the classical concept of motion by replacing from the very start the ordinary kinematical and mechanical quantities by symbols, which refer directly to the individual processes demanded by the quantum postulate. This was accomplished by substituting for the Fourier development of a classical mechanical quantity a matrix scheme, the elements of which symbolise purely harmonic vibrations and are associated with the possible transitions between stationary states. **By requiring that the frequencies ascribed to the elements must always obey the combination principle for spectral lines, Heisenberg could introduce simple rules of calculation for the symbols, which permit a direct quantum theoretical transcription of the fundamental equations of classical mechanics.** This ingenious attack on the dynamical problem of atomic theory proved itself from the beginning to be an exceedingly powerful and fertile method for interpreting quantitatively the experimental results. **Through the work of Born and Jordan as well as of Dirac, the theory was given a formulation which can compete with classical mechanics as regards generality and consistency.** Especially the element characteristic of the quantum theory, Planck's constant, appears explicitly only in the algorithms to which the symbols, the so-called matrices, are subjected. In fact, matrices, which represent canonically conjugated variables in the sense of the Hamiltonian equations, do not obey the commutative law of multiplication, but two such quantities, q and p, have to fulfil the exchange rule $$pq - qp = \sqrt{-1}\frac{h}{2\pi} \quad (3)$$	众所周知，新的发展是从海森堡的一篇奠基性的论文开始的，他成功地从经典运动的概念中解放出来，从一开始就用符号代替了普通的运动学和力学量，这些符号直接对应量子假设所要求的单个过程。这是通过用矩阵方案取代经典力学量的傅里叶展开实现的。矩阵的各个矩阵元象征着纯粹的简谐振动，并与定态之间的可能跃迁相关联。**通过要求矩阵元对应的频率必须始终服从谱线的组合原理，海森堡得以引入适用于这些符号的简单计算规则，从而允许将经典力学基本方程直接转写成量子理论的形式。**这种对原子理论动力学问题的巧妙切入，从一开始就被证明是一种极其强大且富有成效(可以对实验结果进行定量解释)的方法。**通过玻恩、约当和狄拉克的工作，这一理论得到了在普适性和一致性方面堪与经典力学相媲美的系统表达。**特别是量子理论的特征元素——普朗克常数，只在符号，即所谓的矩阵所服从的运算中明确出现。事实上，在哈密顿方程意义上表示正则共轭变量的矩阵并不服从乘法交换律，两个这样的量q和p必须满足对易关系， $$pq - qp = \sqrt{-1}\frac{h}{2\pi} \quad (3)$$

Indeed, this exchange relation expresses strikingly the symbolical character of the matrix formulation of the quantum theory. **The matrix theory has often been called a calculus with directly observable quantities.** It must be remembered, however, that the procedure described is limited just to those problems, in which in applying the quantum postulate the space-time description may largely be disregarded, and the question of observation in the proper sense therefore placed in the background.	的确，这种对易关系鲜明地体现了量子理论矩阵表述的符号特征。**矩阵理论经常被称为直接可观测量的演算工具。**然而，必须记住，所描述的过程仅限于那些在应用量子假设时，时空描述在很大程度上可被忽略、因而观测问题在严格意义上被置于次要地位的问题。
In pursuing further the correspondence of the quantum laws with classical mechanics, the stress placed on the statistical character of the quantum theoretical description, which is brought in by the quantum postulate, has been of fundamental importance. Here the generalisation of the symbolical method made by Dirac and Jordan represented a great progress by making possible the operation with matrices, **which are not arranged according to the stationary states, but where the possible values of any set of variables may appear as indices of the matrix elements.** In analogy to the interpretation considered in the original form of the theory of the 'diagonal elements' connected only with a single stationary state, as time averages of the quantity to be represented, the general transformation theory of matrices permits the representation of such averages of a mechanical quantity, in the calculation of which any set of variables characterising the 'state' of the system have given values, while the canonically conjugated variables are allowed to take all possible values. On the basis of the procedure developed by these authors and in close connexion with ideas of Born and Pauli, Heisenberg has in the paper already cited above attempted a closer analysis of the physical content of the quantum theory, especially in view of the apparently paradoxical character of the exchange relation (3). In this connexion he has formulated the relation $$\Delta q \Delta p \sim h \qquad (4)$$ as the general expression for the maximum accuracy with which two canonically conjugated variables can simultaneously be observed. In this way Heisenberg has been able to elucidate many paradoxes appearing in the application of the quantum postulate, and to a large extent to demonstrate the consistency of the symbolic method. In connexion with the complementary nature of the quantum	**在进一步探讨量子定律与经典力学的对应关系时，把重点放在量子理论描述的统计特征上（这是由量子假设引入的）具有根本的重要意义。**在这方面，狄拉克和约当对符号化方法的推广是一个巨大的进步，它使矩阵的运算成为可能，**这些矩阵不是按照定态排列，而是任何一组变量的可能值作为矩阵元素的指标出现。**类似于最初理论中"对角元素"仅与单一定态相关，并被解释为所研究物理量的时间平均值，矩阵的广义变换理论允许表示这样一种平均值，即对系统"状态"的某组变量赋予特定数值，而其正则共轭变量则取遍所有可能值。根据这些学者所发展的方法，并与玻恩和泡利的思想密切联系，海森堡在上面所引用的论文中试图对量子理论的物理内涵进行更深入的分析，尤其是针对对易关系（3）所呈现出的看似矛盾的特性。在这方面，他阐明关系 $$\Delta q \Delta p \sim h \qquad (4)$$ 作为可以同时观测到的两个正则共轭变量的最大精度的一般表达式。这样，海森堡就能够阐明在应用量子假设时出现的许多矛盾，并在很大程度上证明了符号化方法的一致性。结合量子理论描述的互补性，我们必须如前面提到的那样，时刻牢记物理量的定义可能性与观测可能性。正是为讨论这一问题，由薛定谔发展的波动力学方法，正如我们将看到的，已被证明提供了很大帮助。它允许在相互作用问题上广泛应用叠加原理，从而与上述关于辐射和自由粒子的考虑提供了

theoretical description, we must, as already mentioned, constantly keep the possibilities of definition as well as of observation before the mind. For the discussion of just this question the method of wave mechanics developed by Schrödinger has, as we shall see, proved of great help. It permits a general application of the principle of superposition also in the problem of interaction, thus offering an immediate connexion with the above considerations concerning radiation and free particles. Below we shall return to the relation of wave mechanics to the general formulation of the quantum laws by means of the transformation theory of matrices.	直接联系。下面我们将回到波动力学与通过矩阵变换理论实现的量子定律一般表述之间的关系。

5. WAVE MECHANICS AND QUANTUM POSTULATE Already in his first considerations concerning the wave theory of material particles, de Broglie pointed out that the stationary states of an atom may be visualised as an interference effect of the phase wave associated with a bound electron. It is true that this point of view at first did not, as regards quantitative results, lead beyond the earlier methods of quantum theory, to the development of which Sommerfeld has contributed so essentially. Schrödinger, however, succeeded in developing a wave - theoretical method which has opened up new aspects, and has proved to be of decisive importance for the great progress in atomic physics during the last years. Indeed, the proper vibrations of the Schrödinger wave equation have been found to furnish a representation of the stationary states of an atom meeting all requirements. The energy of each state is connected with the corresponding period of vibration according to the general quantum relation (1). Furthermore, the number of nodes in the various characteristic vibrations gives a simple interpretation to the concept of quantum number which was already known from the older methods, but at first did not seem to appear in the matrix formulation. In addition, Schrödinger could associate with the solutions of the wave equation a continuous distribution of charge and current, which, if applied to a characteristic vibration, represents the electrostatic and magnetic properties of an atom in the corresponding stationary state. Similarly, the superposition of two characteristic solutions corresponds to a continuous vibrating distribution of electrical charge, which on classical electrodynamics would give rise to an emission of radiation, illustrating instructively the consequences of the quantum postulate and the correspondence requirement regarding the transition process between two stationary states formulated in matrix mechanics. Another application of the method of Schrödinger, important for the further development, has been made by Born in his investigation of the problem of collisions between atoms and free electric particles. In this connexion he succeeded in obtaining a statistical interpretation of the wave functions, allowing a calculation of the probability of the individual transition processes required by the quantum postulate. This includes a wave-mechanical formulation of the adiabatic principle of	5. 波动力学和量子假设 早在德布罗意对物质粒子波动理论的初步思考中，他就指出，原子的定态可以被视作与束缚电子对应的相位波的干涉效应。诚然，就定量结果而言，这种观点起初并未超越早期的量子理论方法，而索末菲对这些方法的发展做出了极为重要的贡献。然而，薛定谔成功地发展出了一种波动理论方法，这种方法开辟了新的领域，并被证明对于原子物理学在最近几年取得的巨大进步具有决定性的重要意义。事实上，已经发现薛定谔波动方程的本征振动提供了满足所有要求的原子定态的表示。根据一般量子关系(1)，将每个状态的能量与相应的振动周期联系起来。此外，各种特征振动中的节点数量为量子数这一概念提供了一种简单的解释，该概念在旧的方法中已经存在，但起初似乎并未出现在矩阵表述中。此外，薛定谔能够将波动方程的解与连续的电荷和电流分布联系起来，如果将其应用于特征振动，就代表了原子在相应定态下的静电和磁性特性。类似地，两个特征解的叠加对应于电荷的连续振动分布，这在经典电动力学中会导致辐射的发射，从而清晰地展示了量子假设的后果以及矩阵力学中关于两个稳态之间跃迁过程的对应要求。薛定谔方法的另一个重要应用由玻恩在其研究原子与自由电子粒子碰撞问题时提出，这对后续发展具有重要意义。在此背景下，他成功获得了波函数的统计解释，使得能够计算量子假设所要求的个体跃迁过程的概率。这包括埃伦费斯特绝热原理的波动力学表述，其丰硕成果在洪特关于分子形成问题（通过量子隧穿形成共价键）的研究中得到了显著体现。

Ehrenfest, the fertility of which appears strikingly in the promising investigations of Hund on the problem of formation of molecules.	
In view of these results, **Schrödinger has expressed the hope that the development of the wave theory will eventually remove the irrational element expressed by the quantum postulate and open the way for a complete description of atomic phenomena along the line of the classical theories.** In support of this view, Schrödinger, in a recent paper (*Ann. d. Phys.*, **83**, p. 956; 1927), emphasises the fact that the discontinuous exchange of energy between atoms required by the quantum postulate, from the point of view of the wave theory, is replaced by a simple resonance phenomenon. In particular, the idea of individual stationary states would be an illusion and its applicability only an illustration of the resonance mentioned. It must be kept in mind, however, that just in the resonance problem mentioned we are concerned with a closed system which, according to the view presented here, is not accessible to observation. In fact, wave mechanics just as the matrix theory on this view represents a symbolic transcription of the problem of motion of classical mechanics adapted to the requirements of quantum theory and only to be interpreted by an explicit use of the quantum postulate. Indeed, the two formulations of the interaction problem might be said to be complementary in the same sense as the wave and particle idea in the description of the free individuals. The apparent contrast in the utilisation of the energy concept in the two theories is just connected with this difference in the starting-point.	鉴于这些研究结果，**薛定谔表达了如下期望：波动力学理论的发展将最终消除量子假设所表达的非理性元素，并为基于经典理论框架下的原子现象完整描述开辟道路。**为支持这一观点，薛定谔在近期发表的论文(*Ann. d. Phys.*, **83**, p. 956; 1927)中强调，从波动力学的视角来看，量子假设所要求的原子间能量交换的不连续现象，将被一种简单的共振现象所取代。特别值得注意的是，个体定态的概念可能仅是一种假象，其适用性也仅是对上述共振现象的例证说明。然而，必须牢记的是，在上述共振问题中，我们所关注的是一个封闭系统，根据本文所阐述的观点，该系统无法被观测。事实上，正如矩阵理论一样，波动力学在此观点下代表了经典力学运动问题的符号化转换，这一转换适应了量子理论的要求，并且只能通过明确运用量子假设进行解释。确实，可以认为这两种关于相互作用问题的表述在某种意义上具有互补性，正如波和粒子概念在描述自由个体时的互补性一样。这两种理论在能量概念运用上的明显差异，恰恰与它们的出发点不同有关。
The fundamental difficulties opposing a space-time description of a system of particles in interaction appear at once from the inevitability of the superposition principle in the description of the behaviour of individual particles. **Already for a free particle the knowledge of energy and momentum excludes, as we have seen, the exact knowledge of its space-time co-ordinates. This implies that an immediate utilisation of the concept of energy in connexion with the classical idea of the potential energy of the system is excluded.** In the Schrödinger wave equation these difficulties are avoided by replacing the classical expression of the Hamiltonian by a differential	在描述相互作用粒子系统的时空特性时，根本性的困难立即显现，这源于在描述单个粒子行为时不可避免要遵循叠加原理。正如我们所了解的，即使**对于一个自由粒子，准确知道其能量和动量就无法准确知道其时空坐标。这一事实表明，将能量概念与系统势能的经典观念直接关联的运用是被排除的。**在薛定谔波动方程中，这些困难通过将哈密顿量的经典表达式替换为微分算符而得以规避，这一替换是通过特定关系实现的

operator by means of the relation $$p = \sqrt{-1}\frac{h}{2\pi}\frac{\delta}{\delta q} \quad (5)$$ where p denotes a generalised component of momentum and q the canonically conjugated variable. Hereby the negative value of the energy is regarded as conjugated to the time. So far, in the wave equation, time and space as well as energy and momentum are utilised in a purely formal way.	$$p = \sqrt{-1}\frac{h}{2\pi}\frac{\delta}{\delta q} \quad (5)$$ 其中，p 表示广义动量分量，q 为与之正则共轭的变量。在此，能量的负值被视为与时间共轭。到目前为止，在波动方程中，时间与空间以及能量与动量均以纯粹形式化的方式被运用。
The symbolical character of Schrödinger's method appears not only from the circumstance that its simplicity, similarly to that of the matrix theory, depends essentially upon the use of imaginary arithmetic quantities. But above all there can be no question of an immediate connexion with our ordinary conceptions because the 'geometrical' problem represented by the wave equation is associated with the so-called co-ordinate space, the number of dimensions of which is equal to the number of degrees of freedom of the system, and hence in general greater than the number of dimensions of ordinary space. Further, Schrödinger's formulation of the interaction problem, just as the formulation offered by matrix theory, involves a neglect of the finite velocity of propagation of the forces claimed by relativity theory.	薛定谔方法的符号化特征不仅在于与矩阵理论类似的简洁性，本质上依赖于虚数的使用。更重要的是，由于波动方程所代表的“几何”问题与所谓的“坐标”空间相关联，而该空间的维数等于系统的自由度数，因此通常大于普通空间的维数，所以根本无法与我们通常的概念直接联系起来。此外，薛定谔对相互作用问题的表述，正如矩阵理论所提供的表述一样，忽略了相对论所主张的力的传播速度有限这一事实。
On the whole, it would scarcely seem justifiable, in the case of the interaction problem, to demand a visualisation by means of ordinary space-time pictures. In fact, all our knowledge concerning the internal properties of atoms is derived from experiments on their radiation or collision reactions, such that the interpretation of experimental facts ultimately depends on the abstractions of radiation in free space, and free material particles. Hence, our whole space-time view of physical phenomena, as well as the definition of energy and momentum, depends ultimately upon these abstractions. In judging the applications of these auxiliary ideas we should only demand inner consistency, in which connexion special regard has to be paid to the possibilities of definition and observation.	总体而言，在相互作用问题的情况下，要求通过普通的时空图像进行可视化似乎并不合理。事实上，我们关于原子内部性质的所有知识都源自对其辐射或碰撞反应的实验，所以实验事实的解释最终依赖于自由空间中的光子以及自由物质粒子的*抽象概念*。因此，我们对物理现象的整个时空观，以及能量和动量的定义，最终都依赖于这些抽象概念。在评判这些辅助概念的应用时，我们应仅要求内在一致性，其中特别需要关注定义和观测的可能性。
In the characteristic vibrations of Schrödinger's wave equation we have, as mentioned, an adequate representation of the stationary states of an atom	**在薛定谔波动方程的特征振动中，正如前文所述，我们获得了原子定态的充分表征，这使得通过一般量子关系（1）对系统能量**

allowing an unambiguous definition of the energy of the system by means of the general quantum relation (1). This entails, however, that in the interpretation of observations, a fundamental renunciation regarding the space-time description is unavoidable. In fact, **the consistent application of the concept of stationary states excludes, as we shall see, any specification regarding the behaviour of the separate particles in the atom.** In problems where a description of this behaviour is essential, we are bound to use the general solution of the wave equation which is obtained by superposition of characteristic solutions. We meet here with a complementarity of the possibilities of definition quite analogous to that which we have considered earlier in connexion with the properties of light and free material particles. Thus, while the definition of energy and momentum of individuals is attached to the idea of a harmonic elementary wave, every space-time feature of the description of phenomena is, as we have seen, based on a consideration of the interferences taking place inside a group of such elementary waves. Also in the present case the agreement between the possibilities of observation and those of definition can be directly shown.	**进行明确定义成为可能。然而，这必然导致在观测解释中，对时空描述的根本性放弃。**事实上，正如我们将要看到的，**定态概念的一致应用排除了对原子中单个粒子行为的任何具体描述**。在需要描述此类行为的问题中，我们必然要使用通过特征解叠加获得的波动方程的一般解。在此，我们遇到了定义可能性的互补性，这与我们先前在光的性质及自由物质粒子方面所考虑的情况极为相似。因此，尽管个体能量和动量的定义与谐波基本波的概念相关联，但正如我们所看到的，现象描述的每一个时空特征都基于对一组此类基本波内部干涉的考量。在当前情况下，观测可能性与定义可能性之间的一致性亦可直接得到证明。
According to the quantum postulate any observation regarding the behaviour of the electron in the atom will be accompanied by a change in the state of the atom. As stressed by Heisenberg, this change will, in the case of atoms in stationary states of low quantum number, consist in general in the ejection of the electron from the atom. **A description of the 'orbit' of the electron in the atom with the aid of subsequent observations is hence impossible in such a case.** This is connected with the circumstance that from characteristic vibrations with only a few nodes no wave packages can be built up which would even approximately represent the 'motion' of a particle. The complementary nature of the description, however, appears particularly in that the use of observations concerning the behaviour of particles in the atom rests on the possibility of neglecting, during the process of observation, the interaction between the particles, thus regarding them as free. This requires, however, that the duration of the process is short compared with the natural periods of the atom, which again means that the uncertainty in the knowledge of the energy	**根据量子假设，任何关于原子中电子行为的观测都将伴随着原子状态的改变。**正如海森堡所强调的，在低量子数的定态原子中，这种改变通常表现为电子从原子中逸出。因此，**在这种情况下，借助后续观测来描述原子中电子的“轨道”是不可能的。**这与以下事实相关：从仅有少数节点的特征振动中，无法构建出能够近似表示粒子“运动”的波包。但是描述的互补性尤其体现在，利用关于原子中粒子行为的观测，依赖于在观测过程中忽略粒子间相互作用的可能性，从而将它们视为自由粒子。然而，这要求观测过程的持续时间远短于原子的自然周期，这又意味着在过程中传递能量的不确定性远大于相邻定态之间的能量差。

transferred in the process is large compared to the energy differences between neighbouring stationary states.	
In judging the possibilities of observation it must, on the whole, be kept in mind that the wave mechanical solutions can be visualised only in so far as they can be described with the aid of the concept of free particles. Here the difference between classical mechanics and the quantum theoretical treatment of the problem of interaction appears most strikingly. In the former such a restriction is unnecessary, because the 'particles' are here endowed with an immediate 'reality', independently of their being free or bound. This situation is particularly important in connexion with the consistent utilisation of Schrödinger's electric density as a measure of the probability for electrons being present within given space regions of the atom. Remembering the restriction mentioned, this interpretation is seen to be a simple consequence of **the assumption that the probability of the presence of a free electron is expressed by the electric density associated with the wave-field in a similar way to that by which the probability of the presence of a light quantum is given by the energy density of the radiation.**	在判断观测可能性时，总体上必须牢记，波动力学解仅在其能够借助自由粒子概念进行描述的范围内方可被可视化。在此，经典力学与量子理论在处理相互作用问题上的差异最为显著。前者无需此类限制，因为“粒子”在此被赋予了直接的“实在性”，无论其处于自由态或束缚态。这一情形在将薛定谔的电荷密度作为电子在原子给定空间区域内存在概率的度量时尤为重要。考虑到上述限制，这一解释可被视为一个简单推论，即**自由电子存在的概率由与波场相关的电荷密度表达**[14]**，其方式类似于光量子存在的概率由辐射能量密度给出。** [14] 现在我们知道，波函数的模方表示粒子在某处出现的概率密度。其某点的电荷密度即为电荷乘上概率密度。）
As already mentioned, the means for a general consistent utilisation of the classical concepts in the quantum theory have been created through the transformation theory of Dirac and Jordan, by the aid of which Heisenberg has formulated his general uncertainty relation (4). In this theory also the Schrödinger wave equation has obtained an instructive application. In fact, the characteristic solutions of this equation appear as auxiliary functions which define a transformation from matrices with indices representing the energy values of the system to other matrices, the indices of which are the possible values of the space coordinates. It is also of interest in this connexion to mention that Jordan and Klein (*Zeitsch. f. Phys.*, **45**, 751; 1927) have recently arrived at the formulation of the problem of interaction expressed by the Schrödinger wave equation, taking as starting-point the wave representation of individual particles and applying a symbolic method closely related to the deep-going treatment of the radiation problem developed by Dirac from the point of view of the matrix theory, to which we shall return below.	如前所述，通过狄拉克和约当的变换理论，已为在量子理论中普遍一致地应用经典概念提供了方法。借助这一理论，海森堡得以阐述其广义不确定性关系（4）。在该理论中，薛定谔波动方程也获得了具有启发性的应用。事实上，该方程的特征解表现为辅助函数，这些函数定义了从以系统能量值为指标的矩阵到以空间坐标可能值为指标的其他矩阵的变换。值得一提的是，约当和克莱因(*Zeitsch. f. Phys.*, **45**, 751; 1927)近期从单个粒子的波表示出发，并应用与狄拉克从矩阵理论角度深入处理辐射问题密切相关的符号化方法，得出了由薛定谔波动方程表达的相互作用问题的表述。我们将在下文中再次讨论这一观点。

6. REALITY OF STATIONARY STATES	6. 定态的实在性
In the conception of stationary states we are, as mentioned, concerned with a characteristic application of the quantum postulate. By its very nature this conception means **a complete renunciation as regards a time description**. From the point of view taken here, **just this renunciation forms the necessary condition for an unambiguous definition of the energy of the atom**. Moreover, **the conception of a stationary state involves, strictly speaking, the exclusion of all interactions with individuals not belonging to the system. The fact that such a closed system is associated with a particular energy value may be considered as an immediate expression for the claim of causality contained in the theorem of conservation of energy.** This circumstance justifies the assumption of the supra-mechanical stability of the stationary states, according to which the atom, before as well as after an external influence, always will be found in a well-defined state, and which forms the basis for the use of the quantum postulate in problems concerning atomic structure.	在定态的概念中，如前所述，我们关注的是量子假设的一个典型应用。就其本质而言，**定态概念意味着对时间描述的完全放弃**。从本文所持的观点来看，**正是这种放弃构成了原子能量明确定义的必要条件。**此外，严格来说，**定态的概念意味着排除与不属于该系统的个体的所有相互作用。这样一个封闭系统与特定能量值相关联的事实，可以被视为能量守恒定理中所包含的因果性要求的直接表达。**这一情况证明了定态的超机械稳定性[15]这一假设是合理的，根据这一假设，原子在受到外部影响前后，总是会处于一个明确定义的态，这构成了在涉及原子结构的问题中使用量子假设的基础。 [15] 定态的超机械稳定性假设：这一假设是尼尔斯·玻尔在旧量子论中提出的核心概念之一，指的是，定态具有某种超越经典力学规律的内在稳定性；即使在外界扰动（如外加电磁场）作用下，原子系统在扰动前后始终处于某个确定的定态，而不会停留于任意中间态；这种稳定性不能用经典牛顿力学或麦克斯韦电动力学解释，因而称为“超机械”。
In a judgment of the well-known paradoxes which this assumption entails for the description of collision and radiation reactions, it is essential to consider the limitations of the possibilities of definition of the reacting free individuals, which is expressed by relation (2). In fact, if the definition of the energy of the reacting individuals is to be accurate to such a degree as to entitle us to speak of conservation of energy during the reaction, it is necessary, according to this relation, to coordinate to the reaction a time interval long compared to the vibration period associated with the transition process, and connected with the energy difference between the stationary states according to relation (1). This is particularly to be remembered when considering the passage of swiftly moving particles through an atom. According to the ordinary kinematics, the effective duration of such a passage	在评判这一假设对碰撞和辐射反应描述所引发的著名悖论时，必须考虑反应中自由个体定义可能性的局限性，这一局限性由不确定性关系（2）所表述。事实上，若要使反应中个体的能量定义精确到足以让我们在反应过程中讨论能量守恒的程度，根据这一关系，则有必要为该反应匹配一个时间间隔，该间隔需长于与跃迁过程相关的振动周期，该周期通过关系式(1)与定态间的能量差相关联。在考虑快速运动粒子穿过原子时，这一点尤其值得注意。根据经典运动学理论，此类通过的持续时间相较于原子的自然周期将极为短暂，因此似乎无法将能量守恒原理与定态的稳定性假设相协调（参见 *Zeits. f. Phys.*, **34**, 142; 1925）。然而，在波动表示法中，反应时

would be very small as compared with the natural periods of the atom, and it seemed impossible to reconcile the principle of conservation of energy with the assumption of the stability of stationary states (cf. *Zeits. f. Phys.*, **34**, 142; 1925). In the wave representation, however, the time of reaction is immediately connected with the accuracy of the knowledge of the energy of the colliding particle, and hence there can never be the possibility of a contradiction with the law of conservation. In connexion with the discussion of paradoxes of the kind mentioned, Campbell (*Phil. Mag.*, i. 1106; 1926) suggested the view that the conception of time itself may be essentially statistical in nature. From the view advanced here, according to which the foundation of space-time description is offered by the abstraction of free individuals, a fundamental distinction between time and space, however, would seem to be excluded by the relativity requirement. **The singular position of the time in problems concerned with stationary states is, as we have seen, due to the special nature of such problems.**	间与碰撞粒子能量的认知精度直接相关，因此永远不可能与守恒定律相矛盾。在讨论此类悖论时，坎贝尔(*Phil. Mag.,* i. 1106; 1926）提出了一种观点，即时间概念本身在本质上可能是统计性的[16]。从这里提出的视角来看，时空描述的基础是由自由个体的抽象化所提供的，然而，相对论的要求似乎排除了时间与空间之间任何根本性的区分。正如我们所观察到的，**在涉及定态问题的情况下，时间的特殊地位源于此类问题本身要求放弃时间描述的独特性质。** [16] 统计性：在大量实验的统计规律下，能量和时间呈现不确定性关系。
The application of the conception of stationary states demands that in any observation, say by means of collision or radiation reactions, permitting a distinction between different stationary states, we are entitled to disregard the previous history of the atom. The fact that the symbolical quantum theory methods ascribe a particular phase to each stationary state the value of which depends upon the previous history of the atom, would for the first moment seem to contradict the very idea of stationary states. As soon as we are really concerned with a time problem, however, the consideration of a strictly closed system is excluded. The use of simply harmonic proper vibrations in the interpretation of observations means, therefore, only a suitable idealisation which in a more rigorous discussion must always be replaced by a group of harmonic vibrations, distributed over a finite frequency interval. Now, as already mentioned, **it is a general consequence of the superposition principle that it has no sense to co-ordinate a phase value to the group as a whole, in the same manner as may be done for each elementary wave constituting the group.**	应用定态的概念要求，在任何观测中，例如通过碰撞或辐射反应来区分不同的定态，我们有权忽略原子的先前历史。符号化量子理论方法为每个定态赋予特定相位值，该值取决于原子的先前历史，这一事实乍看似乎与定态概念本身相矛盾。然而，一旦我们真正涉及时间问题，严格封闭系统的考虑便被排除在外。因此，在解释观测结果中使用简谐本征振动仅代表一种合适的理想化，在更严格的讨论中，必须始终用分布在有限频率区间内的一组简谐振动来替代。如前所述，**叠加原理的一个普遍结论是，给这样一个波包整体赋予一个相位值（像对组成波包的每个基本波那样）是没有意义的。**
This inobservability of the phase, well known from the theory of optical instruments, is brought out in a particularly	相位的不可观测性，这一在光学仪器理论中广为人知的现象，在讨论斯特恩-格拉赫

simple manner in a discussion of the Stern-Gerlach experiment, so important for the investigation of the properties of single atoms. As pointed out by Heisenberg, atoms with different orientation in the field may only be separated if the deviation of the beam is larger than the diffraction at the slit of the de Broglie waves representing the translational motion of the atoms. This condition means, as a simple calculation shows, that the product of the time of passage of the atom through the field, and the uncertainty due to the finite width of the beam of its energy in the field, is at least equal to the quantum of action. This result was considered by Heisenberg as a support of relation (2) as regards the reciprocal uncertainties of energy and time values. It would seem, however, that here we are not simply dealing with a measurement of the energy of the atom at a given time. But since the period of the proper vibrations of the atom in the field is connected with the total energy by relation (1), we realise that the condition for separability mentioned just means the loss of the phase. This circumstance removes also the apparent contradictions, arising in certain problems concerning the coherence of resonance radiation, which have been discussed frequently, and were also considered by Heisenberg.	实验时以一种特别简洁的方式得以展现，该实验对于研究单个原子的性质至关重要。正如海森堡所指出的，只有当原子束在磁场中的偏转大于代表原子平移运动的德布罗意波在狭缝处的衍射时，才能将不同取向（对应不同磁量子态）的原子分离开来。通过简单计算可知，这一可分离性条件表明：原子穿过场所需的时间，与因波包的空间有限延展导致的其在场中能量的不确定度的乘积，至少等于作用量量子。海森堡认为这一结果支持了关于能量与时间值的不确定性的关系式（2）。然而，此处似乎并非仅仅涉及在某一给定时刻对原子能量的测量。由于原子在场中的本征振动周期与总能量通过关系式（1）相关联，我们认识到可分离性条件恰好意味着相位的丢失。这一情况也消除了在某些涉及共振辐射相干性问题[17]中出现的明显矛盾。这些问题曾被频繁讨论，且海森堡也曾予以考虑。 [17] 共振辐射相干性问题：指 1924–1927 年间围绕原子共振荧光所暴露出的经典波动图像与量子跃迁图像之间的矛盾。
To consider an atom as a closed system, as we have done above, means to neglect the spontaneous emission of radiation which even **in the absence of external influences puts an upper limit to the lifetime of the stationary states.** The fact that this neglect is justified in many applications is connected with the circumstance that the coupling between the atom and the radiation field, which is to be expected on classical electrodynamics, is in general very small compared to the coupling between the particles in the atom. It is, in fact, **possible in a description of the state of an atom to a considerable extent to neglect the reaction of radiation, thus disregarding the unsharpness in the energy values connected with the lifetime of the stationary states** according to relation (2) (cf. *Proc. Camb. Phil. Soc.*, 1924 (Supplement), or *Zeits. f. Phys.*, **13**, 117; 1923). This is the reason why it is possible to draw conclusions concerning the properties of radiation by using classical electrodynamics.	如前所述，将原子视为一个封闭系统，意味着忽略了这样一个事实，**即使在没有任何外部影响的情况下，原子与真空电磁场的耦合也会导致自发辐射，这对定态的寿命设定了上限。**在许多应用中，这种忽略是合理的，这种基于经典电动力学的预期的原子与辐射场之间的耦合，相较于原子内部粒子之间的耦合而言通常非常微弱。事实上，**在对原子状态的描述中，可以在很大程度上忽略辐射的反作用，从而忽略与定态寿命相关的能量值的不确定性，**这符合不确定性关系（2）（参见 *Proc. Camb. Phil. Soc.*, 1924 (Supplement) ,或 *Zeits. f. Phys.*, **13**, 117; 1923）。这就是通过使用经典电动力学可以得出关于辐射性质的结论的原因。

The treatment of the radiation problem by the new quantum theoretical methods meant to begin with just a quantitative formulation of this correspondence consideration. This was the very starting-point of the original considerations of Heisenberg. It may also be mentioned that an instructive analysis of Schrödinger's treatment of the radiation phenomena from the point of view of the correspondence principle has been recently given by Klein (*Zeits. f. Phys.*, **41**, 707; 1927). In the more rigorous form of the theory developed by Dirac (*Proc. Roy. Soc.*, A, **114**, p. 243; 1927) the radiation field itself is included in the closed system under consideration. Thus it became possible in a rational way to take account of the individual character of radiation demanded by the quantum theory and to build up a dispersion theory, in which the final width of the spectral lines is taken into consideration. The renunciation regarding space-time pictures characterising this treatment would seem to offer a striking indication of the complementary character of the quantum theory. This is particularly to be borne in mind in judging the radical departure from the causal description of Nature met with in radiation phenomena, to which we have referred above in connexion with the excitation of spectra.	新量子理论方法对辐射问题的处理，最初旨在建立对应原理的定量表述。这正是海森堡原始思考的起点。值得一提的是，克莱因（*Zeits. f. Phys.,* **41,** 707；1927）最近从对应原理的角度对薛定谔辐射现象的处理进行了富有启发性的分析。在狄拉克（*Proc. Roy. Soc.,* A，**114**，p. 243；1927）发展的更为严格的理论形式中，辐射场本身被纳入所考虑的封闭系统。因此，得以以合理的方式考虑量子理论所要求的辐射个体特性，并建立一种色散理论，其中谱线的自然线宽也被纳入考量。这种处理方式所摒弃的关于时空图像的描述，似乎清楚地表明了量子理论的互补性特征。在判断与自然因果描述的彻底背离的辐射现象时，这一点尤其需要铭记于心。我们之前已在论述光谱激发时提及了这一点。
In view of the asymptotic connexion of atomic properties with classical electrodynamics, demanded by the correspondence principle, the reciprocal exclusion of the conception of stationary states and the description of the behaviour of individual particles in the atom might be regarded as a difficulty. In fact, the connexion in question means that in the limit of large quantum numbers where the relative difference between adjacent stationary states vanishes asymptotically, mechanical pictures of electronic motion may be rationally utilised. It must be emphasised, however, that this connexion cannot be regarded as a gradual transition towards classical theory in the sense that the quantum postulate would lose its significance for high quantum numbers. On the contrary, the conclusions obtained from the correspondence principle with the aid of classical pictures depend just upon the assumptions that the conception of stationary states and of individual transition processes are maintained even in this limit.	鉴于原子性质与经典电动力学之间的渐近联系，这一联系由对应原理所要求，定态概念与对原子中单个粒子行为的描述之间的相互排斥可以被视为一个难题。事实上，这种联系意味着在大量子数极限下，相邻定态之间的相对差异渐近消失，电子运动的经典力学图像可以合理地加以利用。然而，必须强调的是，这种联系不能被视为向经典理论的逐渐过渡，即量子假设对于大量子数会失去其意义。相反，通过经典图像从对应原理得出的结论恰恰依赖于以下假设：即使在极限情况下，定态概念和单个跃迁过程的描述仍然得以保留。
This question offers a particularly instructive example for the	该问题为应用新方法提供了一个极具启发

application of the new methods. As shown by Schrödinger (*Naturwiss.*, **14**, 664; 1926), it is possible, in the limit mentioned, **by superposition of proper vibrations to construct wave groups small in comparison to the 'size' of the atom, the propagation of which indefinitely approaches the classical picture of moving material particles, if the quantum numbers are chosen sufficiently large.** In the special case of a simple harmonic vibrator, he was able to show that such wave groups will keep together even for any length of time, and will oscillate to and fro in a manner corresponding to the classical picture of the motion. This circumstance Schrödinger has regarded as a support of his hope of constructing a pure wave theory without referring to the quantum postulate. As emphasised by Heisenberg, the simplicity of the case of the oscillator, however, is exceptional and intimately connected with the harmonic nature of the corresponding classical motion. Nor is there in this example any possibility for an asymptotical approach towards the problem of free particles. **In general, the wave group will gradually spread over the whole region of the atom, and the 'motion' of a bound electron can only be followed during a number of periods, which is of the order of magnitude of the quantum numbers associated with the proper vibrations.** This question has been more closely investigated in a recent paper by Darwin (*Proc. Roy. Soc.*, A, **117**, 258; 1927), which contains a number of instructive examples of the behaviour of wave groups. From the viewpoint of the matrix theory a treatment of analogous problems has been carried out by Kennard (*Zeits. f. Phys.*, **47**, 326; 1927).	性的范例。正如薛定谔(*Naturwiss.*, **14**, 664; 1926)所展示的，在上述极限情况下，**通过本征振动的叠加，可以构建出比原子“尺寸”更小的波包，若量子数选择得足够大，其传播将无限趋近于经典物质粒子运动的图像。**在简谐振子的特殊情况下，他能够证明此类波包即使在任意长时间内也能保持完整，并以与经典运动图像相对应的方式来回振荡。薛定谔将这一现象视为其构建不依赖量子假设的纯波动理论的希望的有力支撑。然而，正如海森堡所强调的，振子情况的简单性是特殊的，且与相应经典运动的简谐性质密切相关。在这个例子中，同样不存在任何向自由粒子问题渐近靠拢的可能性。**一般而言，波包将逐渐扩散至整个原子区域，束缚电子的“运动”只能在与其本征振动对应的量子数数量级内的若干周期内进行追踪。**这个问题在达尔文近期的一篇论文(*Proc. Roy. Soc.*, A, **117**, 258; 1927)中得到了更深入的研究，该论文包含了许多有关波包行为的启发性示例。从矩阵理论的角度来看，类似问题的处理由肯纳德(*Zeits. f. Phys.*, **47**, 326; 1927)完成。
Here again we meet with the contrast between the wave theory superposition principle and the assumption of the individuality of particles with which we have been concerned already in the case of free particles. At the same time the asymptotical connexion with the classical theory, to which a distinction between free and bound particles is unknown, offers the possibility of a particularly simple illustration of the above considerations regarding the consistent utilisation of the concept of stationary states. As we have seen, the identification of a stationary state by means of collision or radiation reactions implies a gap in the time description, which is at least of the order of magnitude of the periods associated with transitions between stationary	在此我们再次遇到了波动理论的叠加原理与我们之前在自由粒子情况下所关注的粒子个体性假设之间的对比。与此同时，与经典理论的渐近联系（经典理论中不存在自由粒子和束缚粒子之分）为上述关于一贯地运用定态概念的考量提供了一个尤为简单的例证。正如我们所观察到的，通过碰撞或辐射反应来确定定态意味着时间描述上存在一个间隔，其至少与定态之间跃迁的所对应的周期量级相当。现在，在量子数很高的极限情况下，这些周期可以解释为经典轨道的公转周期。因此，我们立刻可以看出，导致确定某一定态的观测与对原子中

states. Now, in the limit of high quantum numbers these periods may be interpreted as periods of revolution. Thus we see at once that no causal connexion can be obtained between observations leading to the fixation of a stationary state and earlier observations on the behaviour of the separate particles in the atom.	单个粒子先前行为的观测之间，并不存在任何因果联系。
Summarising, it might be said that **the concepts of stationary states and individual transition processes within their proper field of application possess just as much or as little 'reality' as the very idea of individual particles.** In both cases we are concerned with a demand of causality complementary to the space-time description, the adequate application of which is limited only by the restricted possibilities of definition and of observation.	综上所述，可以认为，**在各自适用的领域中，定态概念以及个体跃迁过程，与单个粒子的概念一样，具有完全相同的“实在性”。**在这两种情况下，我们所关注的都是与时空描述相补充的因果性要求，而这种要求的恰当应用仅受限于定义和观察的有限可能性。

7. THE PROBLEM OF THE ELEMENTARY PARTICLES	7. 基本粒子问题
When due regard is taken of the complementary feature required by the quantum postulate, it seems, in fact, possible with the aid of the symbolic methods to build up a consistent theory of atomic phenomena, which may be considered as a rational generalisation of the causal space-time description of classical physics. This view does not mean, however, that classical electron theory may be regarded simply as the limiting case of a vanishing quantum of action. Indeed, the connexion of the latter theory with experience is based on assumptions which can scarcely be separated from the group of problems of the quantum theory. A hint in this direction was already given by the well-known difficulties met with in the attempts to account for the individuality of ultimate electrical particles on general mechanical and electrodynamical principles. In this respect also the general relativity theory of gravitation has not fulfilled expectations. A satisfactory solution of the problems touched upon would seem to be possible only by means of a rational quantum-theoretical transcription of the general field theory, in which the ultimate quantum of electricity has found its natural position as an expression of the feature of individuality characterising the quantum theory. Recently Klein (*Zeits. f. Phys.*, **46**, 188; 1927) has directed attention to the possibility of connecting this problem with the five-dimensional unified representation of electromagnetism and gravitation proposed by Kaluza. In fact, the conservation of electricity appears in this theory as an analogue to the conservation theorems for energy and momentum. Just as these concepts are complementary to the space-time description, the appropriateness of the ordinary four-dimensional description as well as its symbolical utilisation in the quantum theory would, as Klein emphasises, seem to depend essentially on the circumstance that in this description electricity always appears in well-defined units, the conjugated fifth dimension being as a consequence not open to observation.	当充分考虑到量子假设所要求的互补性特征时，借助于符号化方法，事实上有可能构建出一套关于原子现象一致性的理论。该理论可以被视为对经典物理学因果时空描述的合理推广。然而，这一观点绝不意味着经典电子理论可被简单地视为作用量量子趋于零的极限情况。事实上，后一种理论与经验事实之间的联系建立在一些假设之上，这些假设几乎无法与量子理论的一堆问题分开。在尝试根据一般力学和电动力学原理解释基本电荷粒子的个体性时所遇到的众所周知的困难，已经暗示了这一方向。在这方面，广义相对论也未能满足预期。要解决所涉及的问题，似乎只有通过对广义场论进行合理的量子理论重构才可能实现。在这种重构中，电荷这一终极量子作为量子理论“个体性”特征的表现，将找到其自然的位置。近期克莱因（*Zeits. f. Phys.*, **46**, 188; 1927）将的注意力转向了将这一问题与卡鲁扎提出的电磁与引力的五维统一理论相联系的可能性。实际上，在该理论中，电荷守恒表现为能量和动量守恒定律的类比。正如这些概念与时空描述相辅相成，克莱因强调，普通四维描述的适用性及其在量子理论中的符号化运用，似乎本质上取决于这样的事实，即在此描述中电荷总是以明确定义的单位出现，从而导致共轭的第五维度无法被观测到。
Quite apart from these unsolved deep-going problems, the classical electron theory up to the present time has been the guide for a further development of the correspondence description in connexion with the idea first advanced by Compton that the ultimate electrical particles, besides their	除了这些尚未解决的深层次问题之外，迄今为止，经典电子理论一直是我们进一步发展对应描述的指导。这种描述与康普顿首先提出的观念密切相连，即基本电荷粒子除了具有质量和电荷外，还具有由作用

mass and charge, are endowed with a magnetic moment due to an angular momentum determined by the quantum of action. This assumption, introduced with striking success by Goudsmit and 1.Thlenbeck into the discussion of the origin of the anomalous Zeeman effect, has proved most fruitful in connexion with the new methods, as shown especially by Heisenberg and Jordan. One might say, indeed, that the hypothesis of the magnetic electron, together with the resonance problem elucidated by Heisenberg (*Zeits. f. Phys.*, **41**, 239; 1927), which occurs in the quantum-theoretical description of the behaviour of atoms with several electrons, have brought the correspondence interpretation of the spectral laws and the periodic system to a certain degree of completion. The principles underlying this attack have even made it possible to draw conclusions regarding the properties of atomic nuclei. Thus Dennison (*Proc. Roy. Soc.*, A, **115**, 483; 1927), in connexion with ideas of Heisenberg and Hund, has succeeded recently in a very interesting way in. showing how the explanation of the specific heat of hydrogen, hitherto beset with difficulties, can be harmonised with the assumption that the proton is endowed with a moment of momentum of the same magnitude as that of the electron. Due to its larger mass, however, a magnetic moment much smaller than that of the electron must be associated with the proton.	量量子决定的角动量所产生的磁矩。这一假设由古德斯密特和乌伦贝克引入到对反常塞曼效应起源的讨论中，并取得了惊人的成功；在与新方法相结合的过程中，更显示出丰硕的成果，尤其如海森堡与约当所展示的那样。可以说，磁性电子假说，连同海森堡阐明的(*Zeits. f. Phys.*, **41**, 239; 1927)在具有多电子原子的量子理论描述中出现的共振问题，已经使光谱规律和周期振动系统的对应解释达到了一定程度的完善。这种研究方法所依据的原理甚至使人们能够对原子核的性质得出结论。因此，丹尼森(*Proc. Roy. Soc.*, A, **115**, 483; 1927)最近以一种非常有趣的方式，结合海森堡和洪特的观点，展示了如何将氢的比热容的解释（此前一直存在诸多难题）和质子具有与电子同量级角动量的假设相协调。然而，由于质子的质量较大，其对应的磁矩必然远小于电子磁矩。
The insufficiency of the methods hitherto developed as concerns the problem of the elementary particles appears in the questions just mentioned from the fact that they do not allow of an unambiguous explanation of the difference in the behaviour of the electric elementary particles and the 'individuals' symbolised through the conception of light quanta expressed in the so-called exclusion. principle formulated by Pauli. In fact, we meet in this principle, so important for the problem of atomic structure as well as for the recent development of statistical theories, with one among several possibilities, each of which fulfils the correspondence requirement. Moreover, the difficulty of satisfying the relativity requirement in quantum theory appears in a particularly striking light in connexion with the problem of the magnetic electron. Indeed, it seemed not possible to bring the promising attempts made by Darwin and Pauli in generalising the new methods to cover this	迄今为止所发展的方法在处理基本粒子问题上的不足，正体现在前文提及的问题中：这些方法无法对带电基本粒子（指费米子）与由光量子概念所表征的“个体”（指玻色子）在行为上的差异，这种差异由泡利提出的所谓不相容原理所体现。事实上，这一原理对原子结构问题及统计理论新发展至关重要，它所呈现的不过是多种可能性中的一种，而所有这些可能性均满足对应原理的要求。此外，在量子理论中满足相对论要求的困难，在与磁性电子问题相关联时表现得尤为突出。先前达尔文和泡利为涵盖此问题所做的将新方法推广的富有希望的尝试，似乎无法自然地同托马斯对相对论运动学所做的、并为解释实验结果所不可或缺的基本考虑相结合。然而，就在最近，狄拉克(*Proc. of the Roy. Soc.*, A, **117**, 610;

problem naturally, in connexion with the relativity kinematical consideration of Thomas so fundamental for the interpretation of experimental results. Quite recently, however, Dirac (*Proc. of the Roy. Soc.*, A, **117**, 610; 1928) has been able successfully to attack the problem of the magnetic electron through a new ingenious extension. of the symbolical method and so to satisfy the relativity requirement without abandoning the agreement with spectral evidence. In this attack not only the imaginary complex quantities appearing in the earlier procedures are involved, but his fundamental equations themselves contain quantities of a still higher degree of complexity, that are represented by matrices.	1928)成功地通过一种新的、巧妙的符号化方法的拓展解决了磁性电子的问题，从而满足了相对论的要求，同时又保持与光谱证据的一致性。在这一工作中，不仅早期理论中出现的虚数被纳入其中，而且其基本方程本身就含有更高阶复杂度的量——它们由矩阵表示。
Already the formulation of the relativity argument implies essentially the union of the space-time co-ordination and the demand of causality characterising the classical theories. In the adaptation of the relativity requirement to the quantum postulate we must therefore be prepared to meet with a renunciation. as to visualisation in the ordinary sense going still further than in the formulation of the quantum laws considered here. **Indeed, we find ourselves here on the very path taken by Einstein of adapting our modes of perception borrowed from the sensations to the gradually deepening knowledge of the laws of Nature. The hindrances met with on this path originate above all in the fact that, so to say, every word in the language refers to our ordinary perception.** In the quantum theory we meet this difficulty at once in the question of the inevitability of the feature of irrationality characterising the quantum postulate. I hope, however, that the idea of complementarity is suited to characterise the situation, which bears a deep-going analogy to the general difficulty in the formation of human ideas, inherent in the distinction between subject and object.	相对论表述已经在本质上蕴含了时空描述和经典理论的因果性要求的结合。因此，在将相对论的要求与量子假设相适应的过程中，我们必须做好准备，接受在通常意义上的直观性方面所做出的比这里所考虑的量子定律的表述还要彻底的放弃。**事实上，我们发现自己正走在爱因斯坦所走过的道路上，即调整我们源自感觉的认知方式，以适应对自然法则逐渐加深的认识。在这条道路上遇到的障碍，首先源于这样一个事实：可以说，语言中的每一个词都指向我们的日常直觉经验**[18]**。**在量子理论中，我们立刻就在“量子假设所必然带来的非理性特征”这一问题上遇到了这一困难。然而，我希望“互补性”这一观念能够恰当地刻画上述处境——它与人类观念形成过程中因主客体区分而固有的普遍困境之间，存在着深刻的类比。 [18] 日常直觉经验：日常经验里形成的、未经科学修正的直觉式世界图景（例如“粒子沿轨迹运动”“同时具有确定位置和动量”这类经验直观）。